An Integrated Imperfect Production-Inventory Model with Lot-Size-Dependent Lead-Time and Quality Control

Oshmita Dey and Anindita Mukherjee

Abstract In this article, an integrated single-vendor single-buyer production-inventory model with stochastic demand and imperfect production process is investigated. The lead-time is assumed to be dependent on the lot-size and a fixed delay due to non-productive times. A methodology is developed to derive the optimal vendor investment required to reduce the defect rate and thereby minimize the total cost of the integrated system. Under the n-shipment policy, an algorithm is proposed so as to minimize the expected integrated total cost and determine the optimal values of the number of shipments, lot-size, safety stock factor, and percentage of defectives. Numerical results are used to illustrate the effect of various parameters on the system.

Keywords Economic order quantity · Integrated model · Imperfect production Process quality · Variable lead-time

1 Introduction

The integrated single-vendor single-buyer production-inventory problem is inspired by the expanding focus on supply chain management which has been proved to be an adequate means by which both the buyer's and the vendor's interest can be benefited simultaneously [8]. A significant amount of literature [1, 9, 11–13, 17, 19] is available in this regard. In the existing literature, it is mostly found that the demand is deterministic and that shortages are not allowed. This was first extended by Ben-Daya and Hariga [3] where the authors assumed the annual customer demand to be stochastic, thus allowing shortages. Since then various researchers [4, 6, 7, 14] and the references therein have extended the stochastic models under various assumptions. However, in most of these works, the production process quality is presumed to be perfect. Even in models with imperfect production, the production process quality is not taken to be a control parameter [2, 10, 15–17, 21–23]. Ouyang et al. [18]

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did consider process quality improvement but neglected the duration of screening. Dey and Giri [5] extended this existing literature by assuming optimal vendor investment in a stochastic single-vendor single-buyer imperfect production-inventory model with non-negligible screening time. But, they assumed the lead-time to be constant. However, in reality, lead-time is usually not a constant and assuming it to be so is an unreal restriction imposed on the model. Recently, Glock [6] developed a model with variable lead-time extending the Ben-Daya and Hariga's model [3] and permitting batch shipments increasing by a fixed factor. Glock [7] further extended this model by studying the alternative methods for reducing the lead-time and its effect on the expected total costs. Ben-Daya and Hariga [3] assumed the lead-time is taken to be proportional to the lot-size produced by the vendor in addition to a fixed delay due to transportation, non-productive time, etc. This makes sense intuitively since, from a practical point of view, lead-time should be considered as a function of the production lot-size [3]. Keeping this argument in mind, a linear relationship between lead-time and lot-size, including non-productive time, is taken into consideration. Thus, in order to make the model more attuned to reality, the present paper extends Dey and Giri's model [5] by assuming the lead-time to be linearly dependent on the production lot-size and non-productive times.

2 The Model

2.1 Notations

D expected demand rate in units per time for non-defective items
P production rate, $p = \frac{1}{P}$
A buyer's ordering cost per order
F transportation cost per delivery
B vendor's setup cost
L lead-time
h_v vendor's holding cost per item per year
h_{b1} buyer's holding cost for defective items per item per year
h_{b2} vendor's holding cost for non-defective items per item per year
s buyer's unit screening cost
x buyer's screening rate
w vendor's unit warranty cost for defective items
y percentage of defective items produced
k safety stock factor
π buyer's shortage cost per item per year
η fractional opportunity cost
δ percentage decrease in defective items per dollar increase in investment

2.2 Assumptions

- Items of a single product are ordered from a single vendor by a single buyer.
- Demand per unit time is normally distributed with mean D and standard deviation σ.
- An order of nQ (non-defective) items is placed by the buyer to the vendor. These items are produced and, on average, transferred to the buyer in n equal sized shipments by the vendor, n being a positive integer.
- The buyer follows the classical (Q, r) continuous review inventory policy.
- It is assumed that the lead-time depends on the lot-size as per the form $L = pQ + b$, where b is the fixed delay due to transportation, non-productive times, etc. The lead-time demand is defined as the demand during the lead-time period. The lead-time demand is normally distributed with mean $D(pQ + b)$ and standard deviation $\sigma\sqrt{pQ + b}$.
- The re-order point r = expected demand during lead-time + safety stock (SS), i.e., $r = D(pQ + b) + k\sigma\sqrt{pQ + b}$, where k is the safety stock factor.
- Shortages are allowed and completely backlogged.
- y_0 $(0 \le y_0 \le 1)$ is the percentage of defective items produced in each batch of size Q.
- The vendor's rate of production of non-defective items is greater than the demand rate, i.e., $P(1 - y_0) > D$.
- The screening rate x is fixed and is greater than the demand rate i.e., $x > D$.
- The vendor incurs a warranty cost for each defective item produced.
- The vendor invests money to improve the production process quality in terms of buying new equipment, improving machine maintenance and repair, worker training, etc. We consider the following logarithmic investment function $I(y)$ [20]:

$$I(y) = \frac{1}{\delta} \ln\left(\frac{y_0}{y}\right)$$

  where δ is the percentage decrease in y per dollar (or any other suitable currency) increase in investment and y_0 is the original percentage of defective items produced prior to investment.

It is assumed that the vendor accepts an order of size nQ for non-defective items from the buyer. The vendor then produces these nQ items all at once, and then, n batches of Q items are delivered each at a regular interval of $Q(1 - y)/D$ units of time on average. Hence, we can say that each ordering cycle is of length $Q(1 - y)/D$, and the complete production cycle is of length $nQ(1 - y)/D$.

2.3 Buyer's Perspective

The buyer is assumed to follow the classical (Q, r) continuous review inventory system. That is, the buyer places an order of Q items to the vendor once the inventory level falls to the re-order point r. The vendor delivers these items after a lead-time $L = pQ + b$. Here, the safety stock factor k is taken to be a decision variable instead of the re-order point r. On receiving the order from the vendor, the buyer inspects the items at a fixed non-negligible screening rate x. The defective items are discovered in each lot, kept in hold separately and returned to the vendor when the next lot of items arrive. Therefore, the buyer incurs two types of holding cost—one for defective items and one for non-defective items [5]. The average inventory level for non-defective items for the buyer (including those defective items which have not yet been detected before the end of the screening time Q/x) is given by Eq. (1) (Fig. 1).

$$\frac{nQ(1-y)}{D}\left[k\sigma\sqrt{pQ+b}+\frac{Q(1-y)}{2}+\frac{DQy}{2x(1-y)}\right] \tag{1}$$

Equivalently, the average inventory level for defective items is given as below:

$$nQ^2y\left[\frac{1-y}{D}-\frac{1}{2x}\right] \tag{2}$$

Thus, the annual expected total cost for the buyer including the ordering cost, shipment cost, holding cost, shortage cost, and screening cost is given as

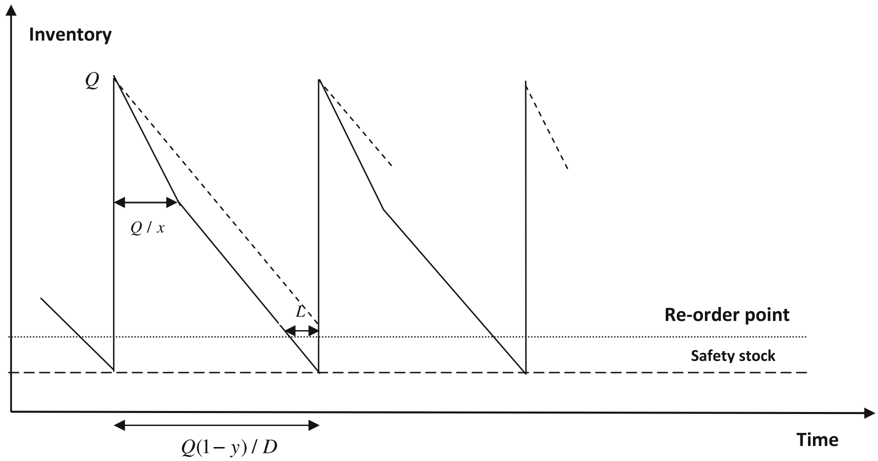

Fig. 1 Inventory of the buyer

$$ETCB(Q,k,n) = \frac{D(A+nF)}{nQ(1-y)} + h_{b1}\left[Qy - \frac{DQy}{2x(1-y)}\right] + h_{b2}\left[k\sigma\sqrt{pQ+b} + \frac{Q(1-y)}{2} + \frac{DQy}{2x(1-y)}\right] + \frac{\pi D\sigma\sqrt{pQ+b}\psi(k)}{Q(1-y)} + \frac{sD}{1-y} \quad (3)$$

where $\psi(k) = \int_k^\infty (z-k)\phi(z)dz$, $\phi(z)$ being the standard normal density function.

2.4 Vendor's Perspective

In the course of the production process, Q items are produced by the vendor in the first instance and then, these items are delivered to the buyer. Thenceforth, a quantity of Q items is delivered by the vendor to the buyer after an interval of every T units of time, where $T = Q(1-y)/D$. This process of delivering the items to the buyer is carried on till the vendor's production run is completed (Fig. 2).

Now, the average inventory holding cost for the vendor [15] is calculated as given below in Eq. (4):

$$EHCV = h_v\frac{Q}{2}\left[n(1 - \frac{Dp}{1-y}) - 1 + \frac{2Dp}{1-y}\right] \quad (4)$$

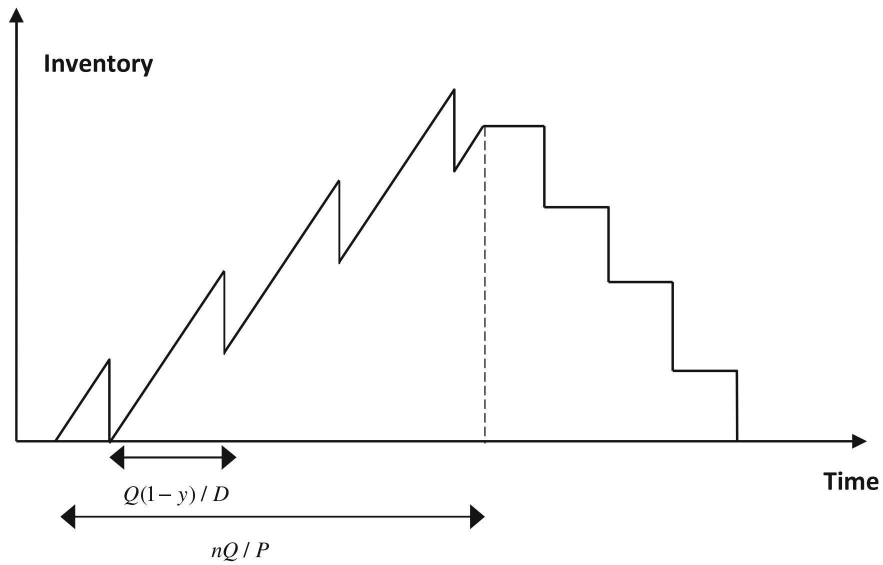

Fig. 2 Inventory of the vendor

Thus, the total cost incurred by the vendor is the sum of the setup cost, holding cost, warranty cost, and investment for reducing the percentage defective items [5] and it is given as

$$ETCV(Q, y, n) = \frac{BD}{nQ(1-y)} + h_v \frac{Q}{2}\left[n(1 - \frac{Dp}{1-y}) - 1 + \frac{2Dp}{1-y}\right] + \frac{wDy}{1-y} + \frac{\eta}{\delta}\ln\left(\frac{y_0}{y}\right) \tag{5}$$

where η is the fractional opportunity cost. It should be taken into account here that the logarithmic investment function considered above is convex in y.

2.5 Integrated System

The total expected annual cost of the integrated system can therefore be expressed as the sum of the buyer's and the vendor's total expected annual costs which is given as below:

$$ETC(Q, y, k, n) = \frac{D(A+B+nF)}{nQ(1-y)} + h_{b1}\left[Qy - \frac{DQy}{2x(1-y)}\right] + h_v \frac{Q}{2}\left[n(1 - \frac{Dp}{1-y}) - 1 + \frac{2Dp}{1-y}\right] + h_{b2}\left[k\sigma\sqrt{pQ+b} + \frac{Q(1-y)}{2} + \frac{DQy}{2x(1-y)}\right] + \frac{\pi D\sigma\sqrt{pQ+b}\psi(k)}{Q(1-y)} + \frac{(s+wy)D}{1-y} + \frac{\eta}{\delta}\ln\left(\frac{y_0}{y}\right) \tag{6}$$

Here, the control parameters are the lot-size Q, the percentage of defectives produced y, the safety stock factor k, and the number of shipments n.

Showing analytically that the expected total cost function, ETC, is convex in all the decision variables Q, y, k and n is not always possible. Nevertheless, the same can be demonstrated numerically. For given fixed values of n (where n is a positive integer) and y ($0 \le y \le y_0 \le 1$), the convexity of total cost function ETC w.r.t Q and k can be easily shown by means of a 3D-graph (Fig. 4). Keeping this potential non-convexity in mind, an iterative algorithm is proposed, in the subsequent section, to derive the optimal values of Q, y, k, and n for which the expected annual total cost for the integrated system ETC is minimized.

3 Solution Procedure

Taking the second-order partial derivative of the total cost function ETC with respect to n, we find,

$$\frac{\partial^2 ETC}{\partial n^2} = \frac{2D(A+B)}{n^3 Q(1-y)} > 0 \qquad \forall \qquad n \geq 1 \tag{7}$$

Thus, from the above equation, we can conclude that ETC is convex in n.

Again, taking the second-order partial derivative of ETC with respect to k and Q, we get,

$$\frac{\partial^2 ETC}{\partial k^2} = \frac{D\sqrt{pQ+b}\sigma\pi\phi(k)}{Q(1-y)} > 0 \tag{8}$$

$$\frac{\partial^2 ETC}{\partial Q^2} = \frac{2DG(n)}{Q^3(1-y)} - \frac{h_{b2}k\sigma p^2}{4(pQ+b)^{\frac{3}{2}}} + \frac{\pi D\sigma\psi(k)}{(1-y)}\left[\frac{2\sqrt{pQ+b}}{Q^3} - \frac{p}{Q^2\sqrt{pQ+b}} - \frac{p^2}{4Q(pQ+b)^{\frac{3}{2}}}\right] > 0 \tag{9}$$

where $G(n) = \frac{A+B+nF}{n}$.

Hence, from Eqs. (8) and (9), ETC is seen to be convex in k and Q for fixed values of n and y ($0 \leq y \leq y_0 \leq 1$). Although y is bounded, it is not possible to prove conclusively that ETC is convex in y. So in order to arrive at an optimal solution, the following procedure is followed:

For fixed value of n, the first derivative of ETC w.r.t k is set to zero. That is,

$$\frac{\partial ETC}{\partial k} = h_{b2} + \frac{\pi D}{Q(1-y)}(F(k)-1) = 0 \tag{10}$$

where $F(\cdot)$ is the cumulative distribution function.

Thus, we have,

$$\overline{F}(k) = \frac{h_{b2}Q(1-y)}{\pi D} \tag{11}$$

where $\overline{F}(\cdot) = 1 - F(\cdot)$.

Next, taking the first derivatives of ETC with respect to Q and y and setting those equal to zero, we get

$$\frac{\partial ETC}{\partial Q} = -\frac{DG(n)}{Q^2(1-y)} + yh_{b1}\{1 - \frac{D}{2x(1-y)}\} + h_{b2}\{\frac{1-y}{2} + \frac{Dy}{2x(1-y)}\} + \frac{h_{b2}k\sigma p}{2\sqrt{pQ+b}} + \frac{h_v}{2}\{-1 + n\big(1 - \frac{Dp}{1-y}\big) + \frac{2Dp}{1-y}\} - \frac{\pi D\sigma\psi(k)}{(1-y)}\left[-\frac{\sqrt{pQ+b}}{Q^2} + \frac{p}{2Q\sqrt{pQ+b}}\right] = 0 \tag{12}$$

and

$$\begin{aligned}\frac{\partial ETC}{\partial y} &= \frac{Dw}{1-y} + \frac{D(s+wy)}{(1-y)^2} - \frac{\eta}{y\delta} + \frac{DG(n)}{Q(1-y)^2} + Qh_{b1}\{1 - \frac{D}{2x(1-y)}\} \\ &- \frac{DQyh_{b1}}{2x(1-y)^2} + h_{b2}\{-\frac{Q}{2} + \frac{DQ}{2x(1-y)} + \frac{DQy}{(1-y)^2}\} \\ &+ \frac{Qh_v}{2}\{\frac{2Dp}{(1-y)^2} - \frac{Dnp}{(1-y)^2}\} - \frac{\pi D\sigma\sqrt{pQ+b\psi(k)}}{Q(1-y)^2} = 0, \end{aligned} \tag{13}$$

respectively.

The algorithm presented by Dey and Giri [5] is modified and used here to derive the optimal solution. It is given as below:

The Algorithm

Step 1: Set $ETC^* = \infty, n = 1$

Step 2: Set $y = y_0$ and $k = 0$ and compute $\psi(k)$ and then compute $Q = Q_0$ using the values of $y_0, k, \psi(k)$ in equation (12)

Step 3: Compute k from (11) using Q_0, y and $\psi(k) = \int_k^\infty (z-k)\phi(z)dz$

Step 4: Compute y from (13) using the values k, Q_0 obtained in the previous step. If $y \geq y_0$, then set $y = y_0$.

Step 5: Compute Q from (12) using the updated values of k, y.
If $|Q - Q_0| \leq \epsilon$, then compute $ETC(Q, k, y, n)$ and go to Step 6.
Else set $Q_0 = Q$ and go back to Step 3.

Step 6: If $ETC^* \geq ETC$, we set $ETC^* = ETC, Q^* = Q, y^* = y, k^* = k, n = n + 1$ and go back to Step 2. Else put $n^* = n - 1$ and stop.
The corresponding values of the control parameters for $n^* = n - 1$ give the optimal solution.

It is to be noted here that we only get a local optimum by adopting the solution procedure mentioned. Since proving analytically that the objective function *ETC* is convex in all control parameters is not possible, we cannot say that the solution obtained above is a global optimum. In order to showcase the effects of the original process quality, the investment option and other model parameters on the optimal decisions, numerical studies are carried out in the following section.

4 Numerical Results and Discussions

For numerical studies, the following data set is considered:
$D = 1000, P = 3200, A = 50, F = 35, K = 400, L = 10/365, h_v = 4, h_{b1} = 6, h_{b2} = 10$, $s = 0.25$, $x = 2152$, $w = 20$, $\pi = 100$, $b = 0.01$, $\sigma = 5$, $y = 0.22$, $\eta = 0.2$, $\delta = 0.0002$

For fixed values of Q, k, n, it is shown that the total expected cost function *ETC* is convex in y $(0 \leq y \leq y_0)$ (Fig. 3). For fixed values of n, y, the convexity of *ETC* w.r.t.Q, k is shown in Fig. 4.

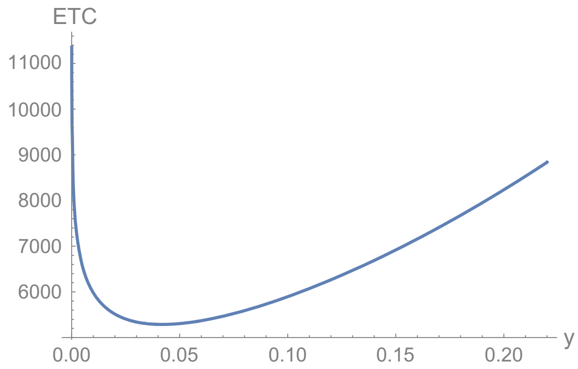

Fig. 3 ETC w.r.ty

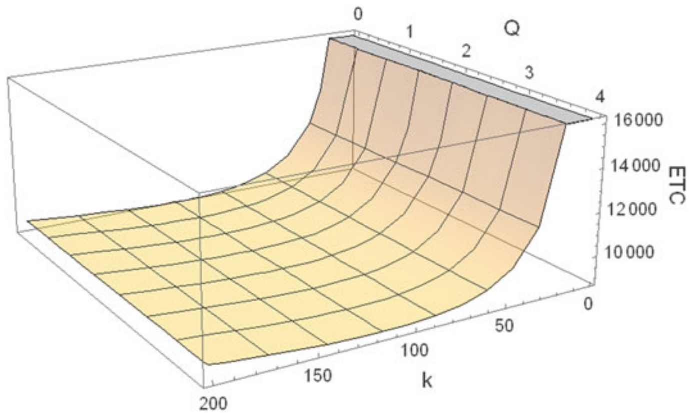

Fig. 4 ETC w.r.t Q, k

Table 1 depicts that the increase in the warranty cost w paid by the vendor results in an increase in the optimal total cost incurred by the supply chain. Also, with an increase in warranty cost, we find a decrease in the optimal value of the percentage of defective items. This is intuitively correct since if a higher warranty cost is to be paid by the vendor as a penalty for producing defective items, it would reasonably be beneficial for him if the number of defective items produced reduces considerably. Following the same logic, an increase in the value of b should imply an increase in the total cost incurred as also shown in Table 1.

A significant conclusion that can be reached from Table 2 is that the investment which is made in order to improve the production process quality is not independent of the original quality. That is, the necessity of an investment and the extent of it being beneficial is decided by the original production process quality. This is evident from Table 2 which clearly shows that investment to improve the production process quality is not needed when the original percentage of defectives is very low.

Table 1 Effect of parameters w and b

		Q^*	n^*	y^*	ETC^*
w	20	86.42	7	0.043	5213.31
	24	86.10	7	0.037	5378.61
	30	95.05	6	0.030	5584.26
b	0.005	86.38	7	0.043	5211.48
	0.010	86.42	7	0.043	5213.31
	0.100	96.01	6	0.043	5235.53

Table 2 Effect of y_0

y_0	Q^*	n^*	y^*	ETC^*	$I(y^*)$
0.010	86.41	7	0.043	2122.27	0.00
0.040	86.41	7	0.043	3508.57	0.00
0.100	86.42	7	0.043	4424.86	843.63
0.220	86.42	7	0.043	5213.31	1632.09
0.418	86.42	7	0.043	5855.17	2273.93
0.680	86.42	7	0.043	6341.78	2760.05

Table 3 Effect of demand rate

d	Q^*	n^*	y^*	ETC^*	$I(y^*)$
800	84.31	6	0.052	4752.10	1438.87
900	90.09	6	0.047	4993.20	1541.46
1000	86.42	7	0.043	5213.31	1632.09
1100	91.53	7	0.039	5413.49	1716.48
1200	96.60	7	0.037	5598.41	1794.05

However, with an increase in the value of y_0, the amount of investment required to optimize the supply chain also increases noticeably.

Table 3 shows that the production lot-size increases with an increase in demand rate, which is very obvious since the buyer would need to place an order of a larger quantity to satisfy the increase in demand. Also, an increase in the lot-size implies that there is an increase in number of both the defective and non-defective items produced, and consequently, the amount of investment needed to optimize the total cost will also increase. So, an increase in demand causes an increase in the optimal lot-size, the total expected cost incurred, and also the optimal vendor investment amount. All these intuitively correct effects are illustrated numerically.

It is seen from Table 4 that for very small values of y_0, the optimal value of ETC obtained for the two cases—with investment and without investment—differs by a small amount. However, as the percentage of defectives increases in the system, there

Table 4 Effect of investment

y_0	ETC* (with investment)	ETC* (without investment)
0.100	4424.86	5069.14
0.220	5213.31	8873.63
0.418	5855.17	18296.6
0.680	6341.78	48540.7

is a significant increase in the value of *ETC* without investment compared to that of *ETC* with investment. Therefore, it can reasonably be concluded that making an investment turns out to be significantly profitable for the supply chain as a whole, especially when the percentage of defectives produced is high.

5 Concluding Remarks

An attempt is made in this paper to analyze the problem of variable lead-time for an integrated single-vendor single-buyer imperfect production-inventory model under optimal vendor investment. It is shown that, as in the case of constant lead-time, for the variable lead-time model also, the investment by the vendor helps in reducing the production yield rate of non-defective items. Further, in case of the vendor making such an investment, the integrated system is better optimized in terms of minimizing the joint expected annual total cost. As a scope of future research, the variable lead-time may be assumed to be controllable. Also, setup cost reduction, inspection errors, variable shipment size, multiple buyers, etc., may also be included.

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