An Integrated Imperfect Production–Inventory Model with Optimal Vendor Investment and Backorder Price Discount

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An Integrated Imperfect Production–Inventory Model with Optimal Vendor Investment and Backorder Price Discount

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Abstract In this article, an integrated single-vendor single-buyer imperfect production–inventory model in which the vendor makes investment for process quality improvement and the buyer offers price discounts for backorders is studied. It is assumed that the buyer follows a continuous review policy with lot-size-dependent lead-time and a mixture of backorders and lost sales. Under the n-shipment policy, the expected annual total cost of the integrated system is derived. An algorithm is developed to determine numerically the optimal decisions of the model. A numerical example is taken to illustrate the developed model and to examine the sensitivity of the key parameters of the model. Some managerial insights are also provided.

Keywords Inventory · Integrated model · Defective items · Backorder price discount

1 Introduction

The problem of a single-vendor single-buyer inventory model and its various extensions have been an area of interest among the practitioners of operations research for quite some time, and a significant amount of literature is available in this regard. For instance, Goyal in [7] was among the first researchers to investigate an integrated inventory model for a single-vendor single-buyer system. Banerjee in [1] generalized the previous model and presented a joint economic lot-size model with the vendor producing items on a lot-for-lot basis. Goyal in [8] further extended this model by relaxing the assumptions of the lot-for-lot policy. Ha and Kim in [10] further generalized this model and developed an integrated lot-splitting model to

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facilitate multiple shipment in small lots. Hill in [11] proposed a more generalized batching and shipment policy. Pa and Yang in [21] extended the model developed by Goyal in [8] by considering lead-time to be a decision variable. Pa and Yang in [24] developed a model where deterministic variable lead-time and quality improvement were considered. Ben-Daya and Hariga in [3] extended the literature on single-vendor single-buyer integrated inventory model by assuming stochastic demand and thereby allowing shortages. Since then a lot of researchers, viz. Hsiao [12], Glock [5, 6], have also made significant contributions in this area. However, in most of these cases, process quality and its effect on the production shipment schedule from the vendor to the buyer were not considered. But, in real-life situations, the production process is generally not perfect. And assuming it to be so is an unrealistic restriction imposed on the model. Porteus [22] was the first to introduce the concept of process quality control and introduced the logarithmic investment function for this purpose. Rosenblatt and Lee in [23] extended this model to the EPQ model. Since then the concept of process quality improvement has been a focal area of interest among researchers. For instance, Zhang and Gerchak in [25] considered a joint lot sizing and shipment policy where the percentage of defectives units is assumed to be random, and that at the end of the screening period, the defective items are returned as a single batch. Ben-Daya and Hariga in [2] developed an economic lot scheduling problem assuming imperfect production process. A practical approach to the EPQ model with imperfect items was proposed by Goyal, Cardenas-Barron in [9]. Huang in [13] developed an integrated vendor–buyer inventory model for items with imperfect quality, where it is assumed that the number of defective items follows a given distribution. Ouyang et al. in [19] also investigated an integrated model with imperfect production process. Lin in [18] considered an integrated supply chain inventory model with imperfect quality items, controllable lead-time, and distribution-free demand. Recently, Dey and Giri in [4] extended existing literature on integrated models by considering vendor investment for process quality improvement in a stochastic integrated inventory model, thereby allowing the percentage of defective items produced to be a control parameter. However, in this model, the shortages were assumed to be completely backlogged. However, it is seen in most cases that, when shortage occurs, while some customers may be willing to wait for their order to arrive at a later stage (backorders), some may be unwilling to do so and may wish to take their orders elsewhere (lost sales). But, incurring lost sales reflect poorly on the customer's goodwill. One way of preventing the percentage of lost sales is to offer a backorder price discount to motivate customers to wait for their orders to arrive with the next batch instead of taking them elsewhere. That is, the buyer may offer the backordered items at a discounted price so as to try and increase the proportion of backorders vis-a-vis lost sales. For instance, an integrated supply chain vendor–buyer model was proposed by Lin [17] where backorder price discount and effective investment required to reduce the ordering cost were considered. Jaggi and Arneja in [14] investigated the periodic review inventory model with backorder price discounts where shortages were partially backlogged. Pal and Chandra in [20] studied a periodic review inventory model with stock-dependent demand, permissible delay in payment, and backorder price discount. Ahamed in [15] developed the lot-size decision for stochastic vendor–buyer system with quantity discount and partial backorder. Jindal and Solanki in [16] studied an integrated supply chain inventory model with quality improvement involving controllable lead-time and backorder price discounts.

Keeping these various issues in mind, an attempt is made to extend existing literature on single-vendor single-buyer integrated inventory models by developing an integrated inventory model with optimal vendor investment and backorder price discount. The model is also investigated under both constant and lot-size-dependent lead-time.

The rest of the paper is organized as follows: In Sect. 2, the proposed model is formulated, and the solution procedure is outlined in Sect. 3. Section 4 illustrates the developed model with the help of numerical examples. The paper is concluded in Sect. 5 with some remarks and future research directions.

2 Model Development

2.1 Notations and Assumptions

The following notations are used for developing the proposed model:

D: Expected demand rate for non-defective items
P: Production rate for the vendor ($P = 1/p$)
A: Ordering cost per order for the buyer
F: Transportation cost per delivery
B: Setup cost for the vendor
L: Lead-time
Q: Order quantity
h_v: Holding cost per item per unit time for the vendor
h_{b1}: Holding cost for defective items per unit time for the buyer
h_{b2}: Holding cost for non-defective items per unit time for the buyer
n: The number of shipments per production run from the vendor to the buyer
r: Reorder point
s: Screening cost per unit item for the buyer
x: Buyer's screening rate
w: Warranty cost per unit defective item for the vendor
k: Safety stock factor
y: Percentage of defective items produced
η: Fractional annual opportunity cost
δ: Percentage decrease in defective items per dollar increase in investment
β: Fraction of the shortage that will be backordered at the buyer's end ($0 \leq \beta < 1$)
β_0: Upper bound of the backorder ratio, ($0 \leq \beta \leq \beta_0 \leq 1$)
π_x: Unit backorder price discount

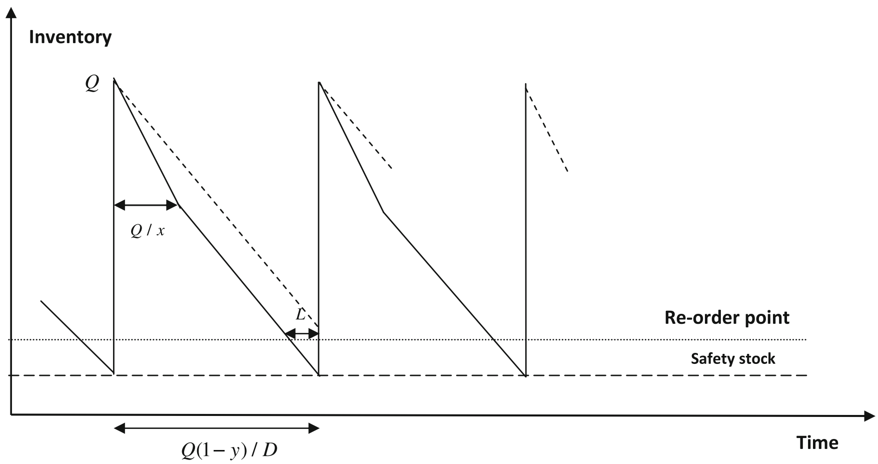

Fig. 1 Inventory of the buyer

π_0: Marginal profit per unit
X: Lead-time demand.

The model is based on the following assumptions (Figs. 1 and 2):

- A single vendor provides single type of item to a single buyer.
- Demand per unit time is normally distributed with mean D and standard deviation σ.
- The buyer orders quantity of nQ items to the vendor which then the vendor produces at one go and delivers in n equal-sized shipments of Q items each to the

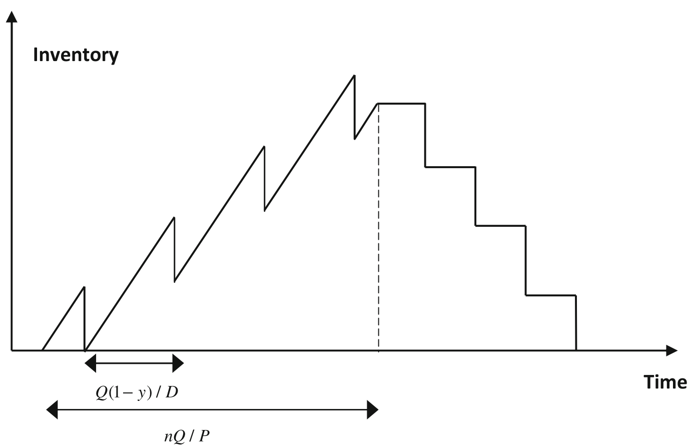

Fig. 2 Inventory of the vendor

buyer. The production rate of non-defective items is assumed to be greater than the demand rate, i.e., $P(1-y) > D$.
- The buyer follows the (Q, r) continuous review policy with constant lead-time and partial backlogging. The case of lot-size-dependent lead-time is also investigated.
- Percentage of backorders is β.
- In case of shortage, the buyer provides a price discount to the customers to encourage them to wait for their orders to arrive with the next batch.
- Lead-time L is constant, and the lead-time demand is normally distributed with mean DL and standard deviation $\sigma\sqrt{L}$.
- The reorder point r = expected demand during lead-time + safety stock (SS), i.e., $r = DL + k\sigma\sqrt{L}$, where k is the safety stock factor.
- y $(0 \le y < 1)$ is the percentage of defective items produced in each lot-size Q.
- The length of vendor's production cycle is $nQ(1-y)/D$, and the length of buyer's ordering cycle is $Q(1-y)/D$.
- The screening rate x is fixed and is greater than the demand rate, i.e., $x > D$.
- The vendor pays a warranty cost for each defective item produced.
- The marginal profit per unit is same as the cost of lost demand (opportunity cost) per unit.
- Investment is made by the vendor in order to improve the production process quality in terms of buying new equipment, worker training, improving machine maintenance and repair, etc. The logarithmic investment function $I(y)$ given by Porteus in [22] is assumed as $I(y) = \frac{1}{\delta}\ln\left(\frac{y_0}{y}\right)$, where δ is the percentage decrease in y for a dollar increase in investment and y_0 is the percentage of defective items produced prior to investment.

2.2 Buyer's Perspective

Assuming the lead-time demand X to be a random variable with associated pdf $f_x(X)$, the expected shortage at the end of a cycle is given by

$$E(X-r)^+ = \int_r^\infty (x-r) f_x(X) dx \tag{1}$$

Since β is assumed to be the percentage of backorders, therefore the expected number of backorders per cycle is $\beta E(X-r)^+$. Thus, the expected stockout cost per unit time is

$$\frac{D}{Q(1-y)}\Big[\pi_x\beta + \pi_0(1-\beta)\Big]E(X-r)^+ \tag{2}$$

Further, the backorder ratio, β, is taken to be a control parameter and is assumed to be in proportion to the unit price discount offered by the buyer. That is, $\beta = (\beta_0\pi_x)/\pi_0$, where $0 \le \beta_0 < 1$ and $0 \le \pi_x \le \pi_0$.

Therefore, the backorder price discount offered by the buyer for each unit of the item, π_x, may be treated as a decision variable in place of the backorder rate, β. Hence, the expected stockout cost per unit time may be written as

$$\frac{D}{Q(1-y)}\left[(\beta_0\pi_x^2/\pi_0)+\pi_0-\beta_0\pi_x\right]E(X-r)^+ \tag{3}$$

Also, the expected net inventory level for non-defective items (just before the arrival of an order) is

$$r-DL+(1-\beta)E(X-r)^+ \tag{4}$$

And, the expected net inventory level for non-defective items (just after the shipment) is

$$Q(1-y)+r-DL+(1-\beta)E(X-r)^+ \tag{5}$$

Thus, the average non-defective inventory per cycle is given by:

$$\frac{Q(1-y)}{2}+r-DL+(1-\beta)E(X-r)^+ \tag{6}$$

Further, on the arrival of the order, the buyer inspects the items at a fixed screening rate x. Here, as is suggested by Dey and Giri in [4], it is assumed that the screening process is nondestructive and error-free and that the buyer incurs two types of holding costs—for defective items and non-defective items (including those defective items which have not yet been detected before the end of the screening time). So, the average inventory for non-defective items is

$$\frac{nQ(1-y)}{D}\left[k\sigma\sqrt{L}+\frac{Q(1-y)}{2}+\frac{DQy}{2x(1-y)}+(1-\beta)E(X-r)^+\right] \tag{7}$$

where

$$r=DL+k\sigma\sqrt{L} \tag{8}$$

Similarly, the average inventory level for defective items is given by [4]

$$nQ^2y[\frac{(1-y)}{D}-\frac{1}{2x}] \tag{9}$$

The resulting expected annual total cost for the buyer, including the shipment cost, ordering cost, inventory holding cost, shortage cost, and screening cost is, therefore, given by

$$ETCB(Q,k,n,y,\pi_x) = \frac{D(A+nF)}{nQ(1-y)} + h_{b1}\left(Qy - \frac{DQy}{2x(1-y)}\right) + h_{b2}\left[k\sigma\sqrt{L} + \frac{Q(1-y)}{2} + (1-\frac{\beta_0\pi_x}{\pi_0})E(X-r)^+ + \frac{DQy}{2x(1-y)}\right] + \frac{D}{Q(1-y)}\left[\frac{\beta_0\pi_x^2}{\pi_0} + \pi_0 - \beta_0\pi_x\right]E(X-r)^+ + \frac{sD}{1-y} \quad (10)$$

where $0 \leq \pi_x \leq \pi_0$, $0 < y \leq y_0$, and $E(X-r)^+$ is the expected demand shortage at the end of the cycle. Since $r = DL + k\sigma\sqrt{L}$, then the expected demand shortage at the end of the cycle is obtained as

$$E(X-r)^+ = \sigma\sqrt{L}\psi(k) \quad (11)$$

where $\psi(k) \equiv \phi(k) - k[1-\phi(k)] > 0$.

Here, $\psi(k)$ and $\phi(k)$ denote the standard normal probability density function and distribution function, respectively. Putting these values in (11), the cost expression for the buyer is obtained as

$$ETCB(Q,k,n,y,\pi_x) = \frac{D(A+nF)}{nQ(1-y)} + h_{b1}\left(Qy - \frac{DQy}{2x(1-y)}\right) + \frac{sD}{1-y} + h_{b2}\left[k\sigma\sqrt{L} + \frac{Q(1-y)}{2} + (1-\frac{\beta_0\pi_x}{\pi_0})\sigma\sqrt{L}\psi(k) + \frac{DQy}{2x(1-y)}\right] + \frac{D}{Q(1-y)}\left[\pi_0 - \beta_0\pi_x + \frac{\beta_0\pi_x^2}{\pi_0}\right]\sigma\sqrt{L}\psi(k) \quad (12)$$

where $0 < y \leq y_0$, $0 \leq \pi_x \leq \pi_0$.

Since $\beta = \frac{\beta_0\pi_x}{\pi_0}$, thus, when $\beta = \beta_0$, we get $\pi_x = \pi_0$ and

$$\pi_0 - \beta_0\pi_x + \frac{\beta_0\pi_x^2}{\pi_0} = \pi_0 = \pi_x \quad (13)$$

Thus, when $\beta = \beta_0$, we get the case where no price discount is offered, then Eq. (12) reduces to

$$ETCB(Q,k,n,y,\pi_x) = \frac{D(A+nF)}{nQ(1-y)} + h_{b1}(Qy - \frac{DQy}{2x(1-y)}) + \frac{sD}{1-y} + h_{b2}[k\sigma\sqrt{L} + \frac{Q(1-y)}{2} + (1-\beta_0)\sigma\sqrt{L}\psi(k) + \frac{DQy}{2x(1-y)}] + \frac{D}{Q(1-y)}\pi_x\sigma\sqrt{L}\psi(k) \quad (14)$$

Further, if $\beta = \beta_0 = 1$ (fully backordered), then $\pi_x = \pi_0$ and the buyer's cost expression reduces to,

$$ETCB(Q,k,n,y,\pi_x) = \frac{D(A+nF)}{nQ(1-y)} + h_{b1}(Qy - \frac{DQy}{2x(1-y)}) + h_{b2}[k\sigma\sqrt{L} + \frac{Q(1-y)}{2} + \frac{DQy}{2x(1-y)}] + \frac{D}{Q(1-y)}\pi_x\sigma\sqrt{L}\psi(k) + \frac{sD}{1-y} \quad (15)$$

In this case, π_x is no longer the marginal price discount but the actual shortage cost is per unit item, which is the same as obtained by Dey and Giri in [4].

2.3 Vendor's Perspective

The vendor produces Q items in the first instance and delivers them to the buyer. After that, the vendor delivers a quantity of Q units to the buyer every T units of time where $T = \frac{Q(1-y)}{D}$. This process continues till the completion of the vendor's production run.

The average inventory holding area for the vendor is as given by Huang in [13]

$$ETCV(Q,n,y) = h_v\frac{Q}{2}\left[(n-1) - (n-2)\frac{Dp}{1-y}\right] \quad (16)$$

The annual expected total cost obtained by the vendor is the sum of the setup cost, holding cost, and warranty cost for the defective items (as given by Huang [13]).

$$ETCV(Q,n,y) = \frac{BD}{nQ(1-y)} + h_v\frac{Q}{2}\left[(n-1) - (n-2)\frac{Dp}{1-y}\right] + \frac{wDy}{1-y} \quad (17)$$

Adding the investment cut for improvement of production process quality as given by Dey and Giri [4], the expected annual total cost of the vendor can be derived as

$$ETCV(Q,k,y,n,\pi_x) = \frac{BD}{nQ(1-y)} + h_v\frac{Q}{2}\left[(n-1) - (n-2)\frac{Dp}{1-y}\right] + \frac{wDy}{1-y} + \frac{\eta}{\delta}\ln\left(\frac{y_0}{y}\right) \quad (18)$$

where η is the fractional opportunity cost.

2.4 Integrated Approach

The expected annual cost of the integrated system is the sum of the vendor's and the buyer's expected annual costs and is therefore obtained as

$$\begin{aligned} ETC(Q, y, k, n, \pi_x) = {} & \frac{D(A + B + nF)}{nQ(1-y)} + h_{b1}\left[Qy - \frac{DQy}{2x(1-y)}\right] + \frac{D(s + wy)}{1-y} \\ & + h_{b2}\left[k\sigma\sqrt{L} + \frac{Q(1-y)}{2} + \frac{DQy}{2x(1-y)} + \left(1 - \frac{\beta_0\pi_x}{\pi_0}\right)\sigma\sqrt{L}\psi(k)\right] \\ & + \frac{\eta}{\delta}\ln\left(\frac{y_0}{y}\right) + \frac{D}{Q(1-y)}G1(\pi_x)\sigma\sqrt{L}\psi(k) \\ & + h_v\frac{Q}{2}\left[(n-1) - (n-2)\frac{Dp}{1-y}\right] \end{aligned} \tag{19}$$

where

$$G1(\pi_x) = (\pi_0 - \beta_0\pi_x + \beta_0\pi_x^2/\pi_0) < 0 \tag{20}$$

(because $\frac{\pi_0}{\pi_x} > \beta_0\left(1 - \frac{\pi_x}{\pi_0}\right) > 0$)

3 Solution Procedure

In the above objective function, the control parameters are Q, y, k, n, and π_x. It is very difficult to show analytically that ETC is a convex function in all the decision variables. However, it can be verified numerically that for given values of n (positive integer), $y (0 < y \leq y_0 < 1)$, and π_x, the total cost function ETC is convex in Q and k (refer Fig. 4). It can also be shown that, all other parameters being known, ETC is convex w.r.t. y (refer Fig. 3). In the following section, a solution procedure is outlined to derive the optimal values of Q, y, k, n, and π_x such that the joint expected annual total cost ETC is minimized.

The total cost function ETC can be shown to be convex in n and π_x, respectively, as

$$\frac{\partial^2 ETC}{\partial n^2} = \frac{2D(A + B)}{n^3 Q(1-y)} > 0 \ \forall\, n \geq 1 \tag{21}$$

$$\frac{\partial^2 ETC}{\partial \pi_x^2} = \frac{2D\sqrt{L}\sigma\beta_0\psi(k)}{Q(1-y)\pi_0} > 0 \tag{22}$$

Keeping in mind the potential non-convexity of the model, an iterative procedure is suggested to derive the optimal values of the control parameters as follows:

For fixed n, the first derivative of ETC with respect to k is put to zero. That is,

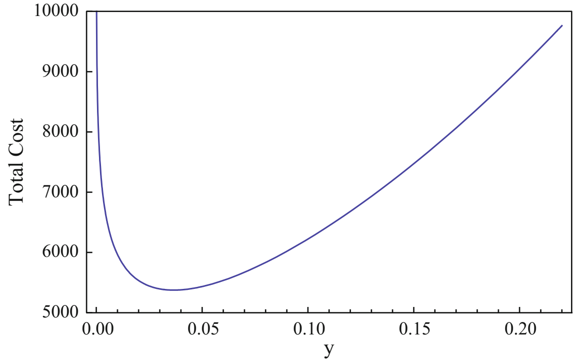

Fig. 3 ETC w.r.ty

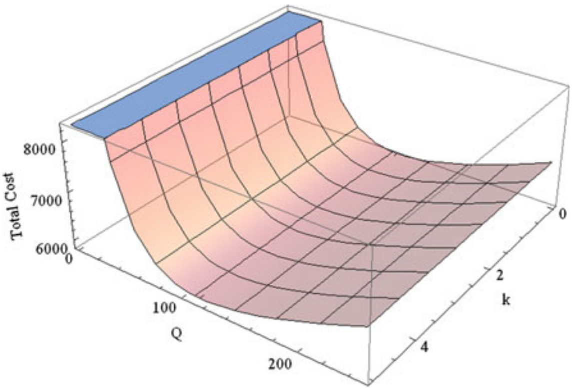

Fig. 4 ETC w.r.t Q, k

$$\frac{\partial ETC}{\partial k} = h_{b2}\left[\sigma\sqrt{L} + \left(1 - \frac{\beta_0\pi_x}{\pi_0}\right)\sigma\sqrt{L}\psi'(k)\right] + \left[\frac{DG1(\pi_x)\sigma\sqrt{L}\psi'(k)}{Q(1-y)}\right] = 0 \qquad (23)$$

Or,

$$\frac{\partial ETC}{\partial k} = h_{b2}\left[\sigma\sqrt{L} + \left(1 - \frac{\beta_0\pi_x}{\pi_0}\right)\sigma\sqrt{L}(F(k)-1)\right] + \frac{DG1(\pi_x)\sigma\sqrt{L}(F(k)-1)}{Q(1-y)} = 0 \qquad (24)$$

where $F(\cdot)$ is the standard normal distribution function.

This implies

$$\overline{F(k)} = \frac{h_{b2}}{H1} = H2(say) \qquad (25)$$

where $H1 = h_{b2}(1 - \beta_0\pi_x/\pi_0) + \frac{DG1(\pi_x)}{Q(1-y)}$ and $\overline{F}(\cdot) = 1 - F(\cdot)$.

Similarly we have

$$\frac{\partial ETC}{\partial Q} = -\frac{DG(n)}{Q^2(1-y)} + yh_{b1}\left(1 - \frac{D}{2x(1-y)}\right) - \frac{D\sqrt(L)G1(\pi_x)\sigma\psi(k)}{Q^2(1-y)} + h_{b2}\left[\frac{1-y}{2} + \frac{Dy}{2x(1-y)}\right] + \frac{h_v}{2}\left[(n-1) - (n-1)\frac{Dp}{1-y}\right] = 0 \tag{26}$$

$$\frac{\partial ETC}{\partial y} = \frac{Dw}{1-y} + \frac{D(t+wy)}{(1-y)^2} - \frac{\eta}{y\delta} + \left[\frac{D\sqrt(L)G1(\pi_x)\sigma\psi(k)}{Q(1-y)^2}\right] + \frac{DG(n)}{Q(1-y)^2} + Qh_{b1}\left[1 - \frac{D}{2x(1-y)} - \frac{Dy}{2x(1-y)^2}\right] - h_{b2}[-\frac{Q}{2} + \frac{DQ}{2x(1-y)} + \frac{DQy}{2x(1-y)^2}] + \frac{Qh_v}{2}\left[\frac{D(n-2)p}{(1-y)^2}\right] = 0 \tag{27}$$

$$\frac{\partial ETC}{\partial \pi_x} = \frac{D\sqrt(L)(-\beta_0 + 2\beta_0\pi_x/\pi_0)\sigma\sqrt(L)\psi(k)}{Q(1-y)} - \left[\frac{\beta_0\sqrt(L)\sigma h_{b2}\psi(k)}{\pi_0}\right] = 0 \tag{28}$$

On simplification, (26) reduces to:

$$Q = \sqrt{\frac{DG(n) + G1(\pi_x)D\sigma\sqrt(L)\psi(k)}{H(n,y)}} \tag{29}$$

where

$$H(n,y) = h_{b1}[y(1-y) - \frac{Dy}{2x}] + h_{b2}[\frac{(1-y)^2}{2} + \frac{Dy}{2x}] + h_v/2 - (n-2)Dp + (n-1)(1-y) \tag{30}$$

From (28), we get

$$\pi_x = \frac{D\pi_0 + Qh_{b2}(1-y)}{2D} \tag{31}$$

It is obvious that the control parameters are not independent of each other. So, to obtain a solution, we adapt the iterative algorithm proposed by Ben-Daya and Hariga in [3] and adapted by Dey and Giri in [4]. First, the algorithm is initiated by setting $y = y_0$, where y_0 is the original percentage of defective items produced. Next, an initial value of Q is calculated by setting the stochastic parameter of Q equal to zero in (29). After calculating Q, the value of k and π_x is obtained using (25) and (31), respectively. Then, using these values, the value of y is updated. This process continues till the minimum cost is determined for the given value of n. The value of n is then updated, and the whole process is repeated till the minimum cost is obtained. The values of the control parameters corresponding to the minimum cost represent the optimal solution. It is to be noted that if the updated value of y is found to be greater than the initial value y_0, then the updated value is rejected. This follows intuitively because the purpose of vendor investment is to improve the production process and failure to do so negates the purpose itself. The method is encapsulated in an algorithm below:

3.1 The Algorithm:

Step 1: Set $ETC^* = \infty$ and $n = 1$;
Step 2: Set $y = y_0$ and compute $Q_0 = \sqrt{DG(n)/H(n, y)}$;
Step 3: Find k from $\psi(k) = \int_k^{\infty}(z - k)\phi(z)dz$ from (25).
Step 4: Compute π_x using (31).
Step 5: Compute y from (27).
Step 6: Compute Q, from (29) using k, y and π_x. If $|Q - Q_0| \leq \epsilon$, compute $ETC(Q, k, y, n, \pi_x)$ and go to step 7. Else set $Q_0 = Q$ and go to step 3;
Step 7: If $ETC^* \geq ETC$, Set $ETC^* = ETC$, $Q^* = Q$, $y^* = y$, $k^* = k$, $n = n + 1$ and go to step 2; Else $n^* = n - 1$ and stop.

The corresponding values of the control parameter for $n^* = n - 1$ give the optimal solution.

Now, partially deriving ETC with respect to w, δ, and y_0 we get

$$\frac{\partial ETC}{\partial w} = \frac{Dy}{1 - y} > 0 \tag{32}$$

$$\frac{\partial ETC}{\partial \delta} = -\frac{\eta}{\delta^2} \ln\left(\frac{y_0}{y}\right) < 0 \tag{33}$$

$$\frac{\partial ETC}{\partial y_0} = \frac{\eta}{y_0 \delta} > 0 \tag{34}$$

From above Eqs. (32)–(34), we observe that ETC increases with an increase in warranty cost w and also with an increase in the original percentage of defective items y_0. For a higher warranty cost, the integrated systems expected total cost will increase and there will also be an increase in the backorder price discount. Also, production of items of very poor quality will result in an increase in the expected total cost of the system. Increasing δ implies that the number of defective items decreases with an increase in the investment amount and, thus, the expected total cost of the system. That is, it costs less to improve the production process quality of the system. To further highlight the effects of process quality, the investment, and other model parameters on the optimal inventory decisions, numerical studies are presented in the following section.

4 Numerical Examples

For numerical studies, we consider the following data set: $D = 1000$, $P = 3200$, $A = 50$, $B = 400$, $F = 35$, $L = 10/365$, $h_v = 4$, $h_{b1} = 6$, $h_{b2} = 10$, $t = 0.25$, $s = 100$, $x = 2152$, $w = 24$, $y = 0.22$, $\eta = 0.2$, $\delta = 0.0002$, $\sigma = 5$, $\pi_0 = 150$, $\beta_0 = 0.2$.

4.1 With Constant Lead-Time

Tables 1, 2, 3, and 4 show the effects of various parameters on the integrated system when the lead-time is constant. (Here z^* denotes π_x^* which is the optimal value of the value of the unit backorder price discount.)

4.2 With Variable Lead-Time

Tables 5, 6, 7, and 8 show the effects of various parameters on the integrated system when the lead-time is taken to be dependent on the lot-size in the form $L = pQ + b$.

Tables 1 and 5 show that the production lot-size increases with an increase in demand rate, which is very obvious since the buyer would need to place an order of a larger quantity to satisfy the increase in demand. Also, there is an increase in the quantity of both defective and non-defective items produced with an increase in

Table 1 Effect of demand rate

D	Q^*	n^*	y^*	ETC^*	$I(y^*)$	k^*	π_x^*
800	84.3442	6	0.044	4911.59	1593.71	2.40639	75.5036
900	90.1529	6	0.040	5153.64	1698.4	2.38026	75.4807
1000	95.8221	6	0.036	5375.09	1792.89	2.35630	75.4616
1100	91.6176	7	0.033	5576.63	1877.08	2.37178	75.4024
1200	96.7118	7	0.031	5762.07	1956.02	2.35073	75.3904

Table 2 Effect of production rate

P	Q^*	n^*	y^*	ETC^*	$I(y^*)$	k^*	π_x^*
2800	87.9689	7	0.036	5335.41	1788.90	2.38794	75.4327
3000	87.1628	7	0.036	5357.04	1790.10	2.39131	75.4198
3200	95.8221	6	0.036	5375.09	1792.89	2.35630	75.4616
3400	95.2388	6	0.036	5389.66	1793.67	2.35855	75.4588
3600	94.7290	6	0.036	5402.55	1794.37	2.36053	75.4563

Table 3 Effect of parameters w and δ

		Q^*	n^*	y^*	ETC^*	$I(y^*)$
w	20	96.154	6	0.042	5210.09	1633.98
	24	95.8221	6	0.036	5375.09	1792.89
	30	95.4785	6	0.030	5580.52	1991.91
δ	0.00020	95.8221	6	0.036	5375.09	1792.89
	0.00024	95.5229	6	0.030	5062.51	1636.14
	0.00030	95.2203	6	0.024	4718.35	1449.84

Table 4 Effect of y_0

y_0	Q^*	n^*	y^*	ETC^*	$I(y^*)$
0.040	95.8221	6	0.036	3670.34	88.1429
0.100	95.8222	6	0.036	4586.63	1004.43
0.220	95.8221	6	0.036	5375.09	1792.89
0.418	95.8221	6	0.036	6016.94	2434.74
0.680	95.8221	6	0.036	6503.55	2921.34

Table 5 Effect of demand rate

D	Q^*	n^*	y^*	ETC^*	$I(y^*)$	k^*	π_x^*
800	75.5015	6	0.044	4915.07	1593.91	2.40789	75.5015
900	89.7808	6	0.040	5157.81	1698.60	2.38173	75.4787
1000	86.1346	7	0.036	5379.64	1791.32	2.39559	75.4149
1100	91.2576	7	0.033	5581.09	1877.27	2.37316	75.4008
1200	96.3325	7	0.031	5767.17	1956.20	2.35211	75.3889

Table 6 Effect of production rate

P	Q^*	n^*	y^*	ETC^*	$I(y^*)$	k^*	π_x^*
2800	87.5897	7	0.037	5340.82	1789.11	2.38946	75.4218
3000	86.8042	7	0.037	5361.61	1790.30	2.39276	75.4181
3200	86.1346	7	0.036	5379.64	1791.21	2.39559	75.4149
3400	85.5569	7	0.036	5395.42	1792.21	2.39804	75.4121
3600	94.3689	6	0.036	5406.08	1794.54	2.36189	75.4546

Table 7 Effect of parameters w and δ

		Q^*	n^*	y^*	ETC^*	$I(y^*)$
w	20	86.4461	7	0.043	5214.35	1632.12
	24	86.1346	7	0.036	5379.64	1791.32
	30	95.0873	6	0.030	5585.32	1992.07
δ	0.00020	86.1346	7	0.036	5379.64	1791.32
	0.00024	95.1318	6	0.030	5067.32	1636.43
	0.00030	94.8299	6	0.024	4723.12	1449.87

Table 8 Effect of y_0

y_0	Q^*	n^*	y^*	ETC^*	$I(y^*)$
0.040	86.1346	7	0.037	3674.90	86.5660
0.100	86.1346	7	0.037	4591.19	1002.86
0.220	86.1346	7	0.036	5379.64	1792.32
0.418	86.1345	7	0.036	6021.50	2433.18
0.680	86.1344	7	0.036	6508.11	2919.78

the lot-size, and consequently, the amount of investment required to optimize the expected annual total cost also increases. Again, we see that an increase in demand causes a decrease in the safety stock factor and the unit backorder price discount also decreases. All these intuitively correct effects are illustrated numerically.

Tables 2 and 6 show that an increase in the production rate simultaneously increases the number of defective and non-defective items produced and hence results in an increase in the amount of investment required to improve the production process quality which in turn increases the expected total cost of the system.

A significant conclusion that can be reached from Tables 4 and 8 is that the original quality and the investment required to improve the production process quality are not independent. That is, the original production process quality decides the necessity of an investment and the extent of it being beneficial to the supply chain. This is evident from Table 1 which clearly shows that investment to improve the production process quality is not needed when the original percentage of defectives is very low. However, the amount of investment required to optimize the integrated system increases noticeably with an increase in the value of y_0.

Tables 3 and 7 depict that the optimal total cost incurred by the supply chain increases with an increase in the warranty cost w paid by the vendor. Also, an increase in warranty cost results in a decrease in the optimal value of the percentage of defective items. This is intuitively correct because, as the vendor is required to pay a higher warranty cost as a penalty for defective item being produced, a considerable decrease in the number of defective items produced would reasonably be favorable for him. Following the same logic, an increase in the value of δ should imply a decrease in the expected annual total cost of the system as also shown in Tables 3 and 7.

5 Managerial Insights and Some Concluding Remarks

The paper presents a single-vendor single-buyer integrated imperfect production–inventory model with vendor investment and backorder price discount. It is assumed that the vendor makes an investment for improving the production process quality. A simple iterative procedure is suggested to obtain an optimal solution of the proposed model in terms of minimizing the total cost incurred. It is shown through numerical examples, under various scenarios, that the production rate of defective items is reduced when the vendor makes an investment. This investment made by the vendor helps in reducing the expected annual total cost, thereby resulting in a better optimized integrated system. It is also evident from the numerical studies that an increased investment is required for an increased demand rate to minimize the total cost. As a scope of future research work, the proposed model can be extended in terms of variable shipment size, inspection errors, controllable lead-time, multiple buyers, etc.

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Further Reading

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