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Application of Kalman Filtering with Bayesian formulation in adaptive sampling

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ABSTRACT

An extensive amount of research is emphasized on survey designs and estimation procedures related to rare and clustered characteristics of a population. Adaptive Sampling design is the most applicable probabilistic technique to estimate the mean or total of the variable of interest, bearing rarity and clustered characteristics. Since rarity is regarded as a time-dependent feature, such surveys need to be organized constantly over time. No studies so far have investigated the effect of time in the estimation context of Adaptive Sampling. This research therefore captures the need to synthesize this periodic information when conducting a survey using Adaptive Sampling design. A recursive process is employed here that improves the estimate of the population parameter from a practical perspective. "Kalman Filtering" is a well known recursive procedure to use past data. Later, statisticians were able to use that Kalman Filtering technique with the Bayesian formulation. This Bayesian approach is proposed to employ here to improve the estimation in the context of Adaptive Sampling design, utilizing the past data. A simulation study is carried out and it is concluded that the suggested approach substantially improves the estimation accuracy.

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1. Introduction

In the case of surveying a population bearing a rarity characteristic, it is difficult to obtain enough units in the sample with the specified rarity criteria. Moreover, traditional sampling methods and related estimation procedures, such as simple random sampling with or without replacement, stratified sampling, sampling with probability proportionate to size, may underestimate the population mean or total due to the absence of a sufficient number of rare units in the sample. A useful method to tackle such populations is discussed in the original three papers of Thompson (1990, 1991a, 1991b, 1992) and known as Adaptive Sampling design for clustered population. Adaptive Sampling design has been recently gaining attention. A huge literature on Adaptive Sampling design is available. A comprehensive review of Turk and Borkowski (2005) covers many advents of this design. Subsequently, Brown and Manly (1998), Salehi and Seber (1997, 2002), Chaudhuri (2000), Chaudhuri, Bose, and Ghosh (2004, Chaudhuri, Bose, and Dihidar 2005), Salehi et al. (2015), Gattone et al. (2016), Salehi and Smith (2021) and many others, strengthened the literature of Adaptive Sampling design. In order to estimate the population parameters, the use of an easily available auxiliary variable in the estimation stage leads to a

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precise estimate. Ratio and regression estimators utilize auxiliary information to obtain better efficiency and information gain. The use of auxiliary information in Adaptive sampling begins with Lee (1998). Chao (2004), Dryver and Chao (2007) suggested ratio estimators for the Adaptive Sampling design. Chaudhuri, Bose, and Dihidar (2005), Pal and Patra (2021, 2023) used auxiliary information to derive generalized regression (greg) estimators with Särndal, Swensson, and Wretman (1992)'s Model-assisted approach.

Further possible way to get a precise estimate is to take into account the information from previous time points in the estimation procedure. Therefore, the present article proposes an improved estimator of the population parameter utilizing the past data, in the context of Adaptive Sampling. The use of time series technique adopting Kalman Filtering (KF) is quite effective and popular for this purpose.

It has been six decades since the Kalman Filtering (KF) technique (Kalman 1960) is introduced for control engineers. This technique has the similarity with two important aspects in Statistics- the linear regression model and time series analysis. With suitable Multivariate Statistics and Bayesian formulation, Meinhold and Singpurwalla (1983) developed the theory of the KF technique for statisticians though it is non-robust. A robust modification was suggested by Meinhold and Singpurwalla (1989) with the tool of Bayesian Statistics. The monograph of Durbin and Koopman (2012) covers most of the relevant theoretical developments of KF technique from Bayesian perspective. Chaudhuri and Maiti (1994), Chaudhuri, Adhikary, and Seal (1997) applied the KF technique in the context of small area estimation. However, most of the research in the KF technique has been done in areas other than survey sampling. It has been applied in various fields such as navigating satellite systems, target tracking objects, digital image processing. Murthy and Federer (2001) emphasized the important applications of the KF technique in a variety of biological domains including Forestry, Hydrology, Fisheries, Agriculture, Environmental monitoring, Medicine, Biotechnology. In medical science, the KF technique has been used in clinical monitoring. It is even used in animal movement modeling. Lu and Zeng (2020) applied the KF technique in order to predict the deformation of rock landslide. In estimating future spread of SARS-Cov-2, Singh et al. (2021) used this techniques, recently.

In the application of this technique, an Observation equation and a System equation are required. The System equation involves two terms that describe the state of nature and measurement error. The Observation equation relates to the observations made in the system over time. Therefore, it is a two steps process - Prediction and Update, for each time point $t = 1, 2, \ldots T$, employing the initial estimate at time $t = 0$. In the prediction step, the KF technique estimates the current state variables, and the outcome of the next measurement is observed and then updated. Under the assumption of error structure, the prediction step is performed first through the Likelihood Function. Bayes rule is applied next to update the estimate.

Rare species are at higher risk of extinction. In order to perpetuate or to provide early warning of endangered species, usually, surveys of such populations are conducted periodically. However, the existing literature of Adaptive Sampling has never been addressed this fact. Thus, this research makes sense to use the KF technique to enhance the accuracy of the estimate of the population parameters when surveys are conducted by Adaptive Sampling design.

The article is organized as follows. Section 2 is designed for an extensive review of Adaptive Sampling design. Section 3 illustrates the actual problem driven by a practical perspective. Due to its practical relevance, this section proposes a way out, following Meinhold and Singpurwalla (1983). This is primarily applied to the Adaptive Sampling design of Thompson (1990). However, Sec. 4 describes the same using the Adaptive Sampling design of Chaudhuri (2000) instead of Thompson (1990). How well the proposed method works is investigated with partially fictitious data and the details are mentioned in Sec. 5. In Sec. 6, the concluding remarks are incorporated.

2. Adaptive sampling

Adaptive Sampling requires an initial sample, employing a sampling design. The basic differences of an initial sample drawing mechanism and the estimation procedures separate Adaptive Sampling designs into two approaches—Thompson's approach and Chaudhuri's approach. Thompson's approach is confined to simple random sampling with or without replacement whereas Chaudhuri's approach considers varying probabilities sampling designs. Two recent treatises, Seber and Salehi (2013) and Chaudhuri (2015), are cited here, considering Thompson's approach and Chaudhuri's approach for subsequent development, respectively.

With the consideration of a finite population $U = (1, 2, \ldots, N)$, let $y = (y_1, y_2, \ldots, y_N)$ be the variable of interest, bearing rare and clustered characteristics. In order to estimate $\tau_Y = \sum_{i=1}^{N} y_i$, the Adaptive Sampling design can be employed. The design and estimation procedures due to Thompson (1990) and Chaudhuri (2000) are stated below.

An initial sample s of size n is selected by a sampling design $P(s)$ and the value of the variable of interest is observed. Whenever the observed ith $(i \in s)$ unit satisfies the pre-defined condition of rarity $y_i > c$, the uniquely defined neighboring units (for example - South, North, East, and West) are searched for further detection of rarity. The neighborhood can be defined in different ways. It may consists of the unit itself and its four or eight adjacent units. Turk and Borkowski (2005) also noted this in their review paper. If some of those units have the rarity characteristic, its neighboring units are also observed. This procedure continues until a unit is detected with no rarity. The neighborhood relation is symmetric. In other words, if the unit i is a neighbor of the unit j, then j is also the neighbor of the unit $i, i \neq j$. All neighboring units related to the initial sampling unit form a cluster. Neighboring units that do not satisfy the condition of rarity are called edge units. Thus, each cluster is bounded by edge units. Eliminating all edge units from a cluster, the rest of the units that meet the predefined criterion of rarity belong to the network of that particular initial unit. It is also noteworthy that if a unit in s does not satisfy the rarity condition, its network consists of that unit only. Thus, for every ith unit, a network $A(i)$ has been formed with the network size m_i. It is necessary to clarify that any unit in $A(i)$ leads to the selection of all the units in $A(i)$.

According to Thompson (1990), the Horvitz–Thompson (HT, Horvitz and Thompson 1952) type unbiased estimator of τ_Y is given by

$$t_{HT} = \sum_{i=1}^{k} \frac{y_i^*}{\alpha_i}; \tag{1}$$

if the initial sample is selected with simple random sampling without replacement.

Here $\alpha_i = 1 - \binom{N-m_i}{n} / \binom{N}{n}$ is the probability of selecting ith network in the sample s and $y_i^* = \sum_{j \in A(i)} y_j$ is the sum of all y values present in ith unit's network. Assuming there are K number of distinct networks in the population U, $k \leq K$ refers to the number of distinct networks in the initial sample s.

An unbiased variance estimator of t_{HT} is

$$v(t_{HT}) = \sum_{i=1}^{k} \sum_{j=1}^{k} \left(\frac{\alpha_{ij} - \alpha_i \alpha_j}{\alpha_{ij} \alpha_i \alpha_j} \right) y_i^* y_j^*; \tag{2}$$

where $\alpha_{ij} = 1 - \left\{ \binom{N-m_i}{n} + \binom{N-m_j}{n} - \binom{N-m_i-m_j}{n} / \binom{N}{n} \right\}$ is the probability of selecting ith and jth networks in the sample s, drawn by simple random sampling without replacement (SRSWOR).

Following Chaudhuri (2000)'s modifications for varying probability sampling design, the Horvitz and Thompson (1952) type unbiased estimator of τ_Y is

$$t_{HT.C} = \sum_{i=1}^{n} \frac{t_i}{\pi_i}; \tag{3}$$

where π_i is the first order inclusion probability of ith unit in the sample s and $t_i = \frac{1}{m_i}\sum_{j\in A(i)} y_j$. It is also observed that $\sum_{i=1}^{N} y_i = \sum_{i=1}^{N} t_i$.

An unbiased variance estimator of $t_{HT.C}$ is

$$v(t_{HT.C}) = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{\pi_i\pi_j - \pi_{ij}}{\pi_{ij}}\right)\left(\frac{t_i}{\pi_i} - \frac{t_j}{\pi_j}\right)^2; \tag{4}$$

where π_{ij} is the second order inclusion probability of ith and jth units in the sample s.

3. Proposed Kalman filtering application in Thompson's adaptive sampling

Let $U(t) = (1, 2, \ldots, N(t))$ be the finite population at the time-point $t = 0, 1, 2, \ldots, T$ and $y(t) = (y_1(t), y_2(t), \ldots, y_{N(t)}(t))$ be the study variable bearing rare and clustered characteristics. The aim is to find an improved estimate of $\tau_Y(t) = \sum_{i=1}^{N(t)} y_i(t)$ at time t, utilizing the past data with Kalman Filtering (KF) technique.

Kalman (1960) introduced a recursive algorithm i.e. KF algorithm which is associated with linear filtering based on the least square method. Later, Meinhold and Singpurwalla (1983) developed the KF technique for statisticians and it is quite easy to understand. Therefore, the computational procedure requires an Observation equation and a System equation. The Observation equation describes the linear relationship between the unobserved quantity and the observed quantity for each time point t. The equation has the structure of a linear regression model. The System equation defines the evolution of the unobservable quantity from the time point $t-1$ to t. Thus, the computational algorithm is mainly based on the recursions, in which the value at the time point t may be calculated from the earlier values for $t-1, t-2, \ldots, 1$. It is noteworthy that the System equation should be linear. However, the Extended Kalman Filtering and Unscented Kalman Filtering (Julier and Uhlmann (1997), Durbin and Koopman (2012)) are designed for the non-linear System equation also. But throughout this study, we concentrate on the linear System equation.

Therefore, to define an Observation equation and a System equation, a suitable model assumption is required. Cassel, Sarndal, and Wretman (1976) and Särndal, Swensson, and Wretman (1992) in their monographs introduced the generalized regression (greg) estimator, postulating a model to describe the relationship between the variable of interest y and an auxiliary variable x. Let $x(t) = (x_1(t), x_2(t), \ldots, x_{N(t)}(t))$ be the known positive valued auxiliary variable, for the time point t.

Now, as described in Sec. 2, an initial sample $s(t)$ of size $n(t)$ is drawn at time t and to capture more rare units, Thompson's Adaptive Sampling steps are followed for each time point t. Defining $A(i;t)$ as the network of ith unit in the initial sample at the time point t, $y_i^*(t) = \sum_{j\in A(i;t)} y_j(t)$ and $x_i^*(t) = \sum_{j\in A(i;t)} x_j(t)$ represent the sum of the y and x values of the units which are in the same network.

To define the Greg estimator of Thompson (1990)'s Adaptive Sampling, let a model be postulated as

$$y_i^*(t) = \beta(t)x_i^*(t) + \varepsilon_i(t)$$

where $\varepsilon_i(t)$'s are assumed to be independent and identically distributed with mean 0 and variance $\sigma^2(t)$.

Then, the Greg estimator of $\tau_Y(t)$ can be written as

$$t_{greg}(t) = \sum_{i=1}^{k(t)} \frac{y_i^*(t)}{\alpha_i(t)} + \left(X(t) - \sum_{i=1}^{k(t)} \frac{x_i^*(t)}{\alpha_i(t)} \right) \hat{\beta}(t); \tag{5}$$

where $k(\mathrm{t})$ is the number of distinct networks in the sample $s(t)$ at the time point t, $X(t) = \sum_{i \in U(t)} x_i^*(t)$, $\hat{\beta}(t) = \frac{\sum_{i=1}^{k(t)} y_i^*(t) x_i^*(t) w_i(t)}{\sum_{i=1}^{k(t)} x_i^{*2}(t) w_i(t)}$ is the regression coefficient and $w_i(t) = \frac{1-\alpha_i(t)}{\alpha_i(t) x_i^*(t)}$.

The above equation can be rewritten as

$$t_{greg}(t) = \sum_{i=1}^{k(t)} \frac{y_i^*(t)}{\alpha_i(t)} \left[1 + \left\{ X(t) - \sum_{i=1}^{k(t)} \frac{x_i^*(t)}{\alpha_i(t)} \right\} \frac{x_i^*(t) w_i(t) \alpha_i(t)}{\sum_{i=1}^{k} x_i^{*2}(t) w_i(t)} \right] = \sum_{i=1}^{k(t)} \frac{y_i^*(t)}{\alpha_i(t)} g_i(t); \tag{6}$$

where $g_i(t) = 1 + \left\{ X(t) - \sum_{i=1}^{k(t)} \frac{x_i^*(t)}{\alpha_i(t)} \right\} \frac{x_i^*(t) w_i(t) \alpha_i(t)}{\sum_{i=1}^{k(t)} x_i^{*2}(t) w_i(t)}$.

Let $e_i(t) = y_i^*(t) - \hat{\beta}(t) x_i^*(t)$ and $F_i(t) = g_i(t) e_i(t)$. Then, an unbiased variance estimator of $t_{greg}(t)$ is

$$v_{greg}(t) = \sum_{i=1}^{k(t)} \sum_{j=1}^{k(t)} \left(\frac{\alpha_{ij}(t)}{\alpha_i(t) \alpha_j(t)} - 1 \right) \frac{F_i(t) F_j(t)}{\alpha_{ij}(t)} \ ; \tag{7}$$

and the variance of $\hat{\beta}(t)$ is given by

$$V\left(\hat{\beta}(t)\right) = V_p E_m\left(\hat{\beta}(t)\right) + E_p V_m\left(\hat{\beta}(t)\right) = \sigma^2(t) E_p \left(\frac{\sum_{i=1}^{k(t)} x_i^{*2}(t) w_i^2(t)}{\left(\sum_{i=1}^{k(t)} x_i^{*2}(t) w_i(t) \right)^2} \right); \tag{8}$$

where E_p, E_m be the design-based, model-based expectation and V_p, V_m be the same for the variance.

However, in the above expression, $\sigma^2(t)$ is unknown. Thus, it can be estimated by an unbiased estimator $\frac{1}{n(t)-1} \sum_{i=1}^{k(t)} e_i^2(t)$.

Therefore, an estimator of $V(\hat{\beta}(t))$ may be taken as

$$\mathrm{A}^*(t) = \left(\frac{1}{n(t) - 1} \sum_{i=1}^{k(t)} e_i^2(t) \right) \left(\frac{\sum_{i=1}^{k(t)} x_i^{*2}(t) w_i^2(t)}{\left(\sum_{i=1}^{k(t)} x_i^{*2}(t) w_i(t) \right)^2} \right) \tag{9}$$

Now, utilizing $v_{greg}(t)$ and $\mathrm{A}^*(t)$ as model parameters, the estimate $t_{greg}(t)$ can be improved. This $t_{greg}(t)$ also depends on the regression coefficient $\beta(t)$ and a known quantity $X(t)$.

Following Meinhold and Singpurwalla (1983), $t_{greg}(t)$ can be updated through $\beta(t)$, using the data on the previous time point.

Therefore, the KF model can be modified as

Observation equation:

$$t_{greg}(t) = \beta(t)X(t) + e(t);\;\; e(t) \sim N\big(0, v_{greg}(t)\big),\;\; t = 1, 2, \ldots, T \tag{10}$$

System equation:

$$\beta(t) = \beta(t-1) + a(t);\; a(t) \sim N\big(0, \mathrm{A}^*(t)\big),\; assuming\;\; e(t),\;\; a(t) are\;\; independent. \tag{11}$$

In Eq. (10), it shows the linear relationship between the Greg estimator $t_{greg}(t)$ and the regression coefficient $\beta(t)$. The evolution of $\beta(t)$ is defined by the System equation (Eq. (11)).

A few notations are needed here for the recursive algorithm. Let $t_{greg}(\underset{\sim}{t}) = (t_{greg}(\underset{\sim}{t}-1), t_{greg}(t));\;\; t = 2, 3, \ldots., T$ and $P(\beta(t)|t_{greg}(\underset{\sim}{t}))$ be a posterior distribution.

Denoting $t_{greg}(t)$ as the initial sample-based estimate, the prior distribution of $\beta(t)$ is $P(\beta(t)|t_{greg}(\underset{\sim}{t}-1))$ and the likelihood of $\beta(t)$ is $P(t_{greg}(t)|\beta(t), t_{greg}(\underset{\sim}{t}-1))$.

Therefore, following Meinhold and Singpurwalla (1983), it can be written as

$$P\left(\beta(t)\middle|t_{greg}(\underset{\sim}{t})\right) \propto P\left(t_{greg}(t)|\beta(t), t_{greg}(\underset{\sim}{t}-1)\right)P(\beta(t)|t_{greg}(\underset{\sim}{t}-1)\Bigg); \tag{12}$$

It is also noteworthy that $\beta(t-1)$ can be summarized by the posterior distribution $P(\beta(t-1)|t_{greg}(\underset{\sim}{t}-1))$ which is here assumed to be followed $N(\varphi(t-1), \Sigma(t-1));\;\; t = 1, 2, \ldots, T.$

The starting value at the time point $t = 0$ is distributed as,

$$\beta(t) \sim N\big(\varphi(0),\;\; \Sigma(0)\big) \;\text{ where }\; \varphi(0) = \frac{t_{greg}(0)}{X(0)} \;\text{ and }\; \Sigma(0) = \frac{v_{greg}(0)}{X^2(0)} \tag{12.1}$$

are the best guesses.

Then, it is found that

$$\beta(t)|t_{greg}(\underset{\sim}{t}-1) \sim N\big(\varphi(t-1), R(t) = \Sigma(t-1) + \mathrm{A}^*(t)\big); \tag{13}$$

which is derived from the system equation defined in Eq. (11).

Therefore, to calculate the posterior distribution defined in Eq. (12) it is necessary to determine the likelihood function of $t_{greg}(t)|\beta(t), t_{greg}(\underset{\sim}{t}-1)$. Let $\Delta(t)$ be the measurement residual which may be written as

$$\Delta(t) = t_{greg}(t) - t^*_{greg}(t) = t_{greg}(t) - \varphi(t-1)X(t). \tag{14}$$

Thus, from the Eq. (12) and Eq. (14) it is also found that the likelihood function $t_{greg}(t)|\beta(t), t_{greg}(\underset{\sim}{t}-1)$ is equivalent to $\Delta(t)|(t_{greg}(\underset{\sim}{t}-1), \beta(t))$.

Using the observation equation Eq. (10) in the Eq. (14), it is found that

$$\Delta(t) = \beta(t)X(t) + e(t) - \varphi(t-1)X(t) = X(t)\big(\beta(t) - \varphi(t-1)\big) + e(t);$$

which yields $E(\Delta(t)|t_{greg}(\underset{\sim}{t}-1), \beta(t)) = X(t)\big(\beta(t) - \varphi(t-1)\big)$. Thus,

$$\Delta(t)|\left(t_{greg}(\underset{\sim}{t}-1), \beta(t)\right) \sim\; N\Big(X(t)\big(\beta(t) - \varphi(t-1)\big), v_{greg}(t)\Big). \tag{15}$$

Therefore, the posterior distribution $\beta(t)|t_{greg}(\underset{\sim}{t})$ becomes $\beta(t)|\Delta(t), t_{greg}(\underset{\sim}{t}-1)$ which can be computed using the property of Bivariate Normal distribution: Suppose $X_1,\; X_2$ follow Bivariate Normal distribution with means $\mu_1,\;\; \mu_2$ and has the covariance matrix $\Sigma =$

$\begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$. Then, the conditional distribution is

$$X_1|X_2 = x_2 \sim N\left(\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(x_2 - \mu_2), \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}\right) \tag{16}$$

Now, replacing X_1, X_2, μ_2 and Σ_{22} by $\Delta(t), \beta(t), \varphi(t-1)$ and $R(t) = \Sigma(t-1) + A^*(t)$, respectively in Eq. (16) and then, comparing it with the mean of the distribution of $\Delta(t)|\beta(t), \underset{\sim}{t_{greg}}(t-1)$ (Eq. (15)), the value of μ_1 and Σ_{12} are computed.

Thus, one may get the following equation

$$\mu_1 + \Sigma_{12}R^{-1}(t)\left(\beta(t) - \varphi(t-1)\right) = X(t)\left(\beta(t) - \varphi(t-1)\right);$$

which leads to the solutions

$$\mu_1 = 0 \text{ and } \Sigma_{12} = X(t)R(t). \tag{17}$$

Similarly, comparing variances, it can be easily seen that $\Sigma_{11} = v_{greg}(t) + X^2(t)R(t)$.
Then, the joint distribution of $\beta(t)$ and $\Delta(t)$, given $\underset{\sim}{t_{greg}}(t-1)$ is

$$\begin{pmatrix} \beta(t) \\ \Delta(t) \end{pmatrix} \Big| \underset{\sim}{t_{greg}}(t-1) \sim N\left(\begin{pmatrix} \varphi(t-1) \\ 0 \end{pmatrix}, \begin{pmatrix} R(t) & X(t)R(t) \\ X(t)R(t) & v_{greg}(t) + X^2(t)R(t) \end{pmatrix}\right) \tag{18}$$

Thus, the posterior distribution becomes,

$$\beta(t)|\underset{\sim}{t_{greg}}(t) \sim N\left(\varphi(t-1) + \frac{R(t)X(t)}{X^2(t)R(t) + v_{greg}(t)}\Delta(t), R(t) - \frac{R^2(t)X^2(t)}{X^2(t)R(t) + v_{greg}(t)}\right) \tag{19}$$

Now, at time point t, the posterior distribution for $\beta(t)$ has the mean and variance

$$\varphi(t) = \varphi(t-1) + \frac{R(t)X(t)}{X^2(t)R(t) + v_{greg}(t)}\Delta(t) \tag{19.1}$$

and

$$\Sigma(t) = R(t) - \frac{R^2(t)X^2(t)}{X^2(t)R(t) + v_{greg}(t)}, \text{ respectively.} \tag{19.2}$$

The term $\frac{R(t)X(t)}{X^2(t)R(t)+v_{greg}(t)}$ is called 'Kalman Gain'. This Kalman gain is used to update the model.

Therefore, the updated regression coefficient is

$$\hat{\beta}_{KF}(t) = \varphi(t-1) + \frac{R(t)X(t)}{X^2(t)R(t) + v_{greg}(t)}\Delta(t) = \varphi(t); t = 1, 2, \ldots, T. \tag{20}$$

and the updated Greg estimator at time point t is

$$t_{greg}^{KF}(t) = \hat{\beta}_{KF}(t)X(t); \tag{21}$$

which has the measurement error

$$\Sigma(t)X^2(t); \tag{22}$$

where $\Sigma(t) = R(t) - \frac{R^2(t)X^2(t)}{X^2(t)R(t)+v_{greg}(t)}$.

4. Extension of the proposed method for varying probabilities sampling design

Chaudhuri (2000) demonstrated necessary modifications of Adaptive Sampling design where an initial sample s of size n is chosen by any varying probabilities sampling designs instead of SRS.

This procedure and the estimation procedure are described in Sec. 2. In Sec. 2, Eq. (3) and Eq. (4) define the unbiased estimator of τ_Y and unbiased variance estimator, respectively.

In order to perform the recursive algorithm of Meinhold and Singpurwalla (1983) with Chaudhuri's adaptive sampling design, a model consideration is required for deriving the Greg estimator at the time-point t.

Let a model be postulated as

$$t_i(t) = \beta^*(t)l_i(t) + \varepsilon_i(t); \;\; t = 1, 2, ., \;\; T$$

where, $t_i(t) = \frac{1}{m_i(t)}\sum_{j\in A(i;t)} y_j(t)$, $l_i(t) = \frac{1}{m_i(t)}\sum_{j\in A(i;t)} x_j(t)$, $L(t) = \sum_{i\in U(t)} l_i(t)$ and as before, $\varepsilon_i(t)$ are assumed to be independently and identically distributed with mean 0 and variance $\sigma^2(t)$ (unknown).

Hence, the Greg estimator of $\tau_Y(t) = \sum_{i=1}^{N(t)} y_i(t)$ is

$$t_{greg.C}(t) = \sum_{i=1}^{n(t)} \frac{t_i(t)}{\pi_i(t)}\left[1 + \left\{L(t) - \sum_{i=1}^{n(t)} \frac{l_i(t)}{\pi_i(t)}\right\}\frac{l_i(t)w_i'(t)\pi_i(t)}{\sum_{i=1}^{n} l_i^2(t)w_i'(t)}\right] = \sum_{i=1}^{n(t)} \frac{t_i(t)}{\pi_i(t)} g_i'(t); \tag{23}$$

where $g_i'(t) = 1 + \left\{L(t) - \sum_{i=1}^{n(t)} \frac{l_i(t)}{\pi_i(t)}\right\}\frac{l_i(t)w_i'(t)\pi_i(t)}{\sum_{i=1}^{n} l_i^2(t)w_i'(t)}$ and $w_i'(t) = \frac{1-\pi_i(t)}{\pi_i(t)l_i(t)}$.

Therefore, the unbiased variance estimator of $t_{greg.C}(t)$ is

$$v_{greg.C}(t) = \sum_{i<j\in s(t)}\sum \left(\frac{\pi_i(t)\pi_j(t) - \pi_{ij}(t)}{\pi_{ij}(t)}\right)\left(\frac{F_i'(t)}{\pi_i} - \frac{F_j'(t)}{\pi_j}\right)^2; \tag{24}$$

where $e_i'(t) = t_i(t) - \widehat{\beta}^*(t)l_i(t)$, $F_i'(t) = g_i'(t)e_i'(t)$ and $\widehat{\beta}^*(t) = \frac{\sum_{i=1}^{n(t)} t_i(t)l_i(t)w_i'(t)}{\sum_{i=1}^{n(t)} l_i^2(t)w_i'(t)}$.

Thus, the KF model can be defined as

The observation equation:

$$t_{greg.c}(t) = \beta^*(t)L(t) + e^*(t); \; e^*(t) \sim N\big(0, v_{greg.C}(t)\big), \;\; t = 1, 2, ..., T \tag{25}$$

and **the system equation:**

$$\beta^*(t) = \beta^*(t-1) + a^*(t); \; a^*(t) \sim N\big(0, \mathrm{A}^{**}(t)\big). \tag{26}$$

Here, $A^{**}(t) = (\frac{1}{n(t)-1}\sum_{i=1}^{n(t)} e_i'^2(t))\frac{\sum_{i=1}^{n(t)} l_i^2(t)w_i'^2(t)}{(\sum_{i=1}^{n(t)} l_i^2(t)w_i'(t))^2}$ is an estimator of $V(\widehat{\beta}^*(t))$. It is also assumed that $e^*(t)$ and $a^*(t)$ are independent.

Using Eqs. (12)–(19), it is found that the predicted regression coefficient is

$$\hat{\beta}_{KF.C}(t) = \varphi^*(t-1) + \frac{R^*(t)\mathrm{L}(t)}{L^2(t)R^*(t) + v_{greg.C}(t)}\Delta^*(t) = \varphi^*(t); \;\; t = 1, 2, ..., T. \tag{27}$$

For convenience, the recursive steps are written as follows,

$$R^*(t) = \Sigma^*(t-1) + \mathrm{A}^{**}(t); \;\; t = 1, 2, ..., T \tag{28}$$

$$\Sigma^*(t) = R^*(t) - \frac{R^{*2}(t)L^2(t)}{L^2(t)R^*(t) + v_{greg.C}(t)}; \;\; t = 1, 2, ..., \;\; T \tag{29}$$

$$t^*_{greg.C}(t) = L(t)\varphi^*(t-1); \;\; t = 1, 2, ..., \;\; T \tag{30}$$

$$\Delta^*(t) = t_{greg.C}(t) - t^*_{greg.C}(t); \;\; t = 1, 2, ..., T \tag{31}$$

$$\varphi^*(t) = \varphi^*(t-1) + \frac{R^*(t)L(t)}{L^2(t)R^*(t) + v_{greg.C}(t)}\Delta^*(t);\ \ t = 1, 2, ..., T \tag{32}$$

The initial value at the time point $t = 0$ for $\beta^*(t)$ is assumed to be distributed as $N(\varphi^*(0), \Sigma^*(0))$ where $\varphi^*(0) = \frac{t_{greg.C}(0)}{L(0)}$ and $\Sigma^*(0) = \frac{v_{greg.C}(0)}{L^2(0)}$.

Therefore, the updated predictor of $t_{greg.C}(t)$ is

$$t^{KF}_{greg.C}(t) = \hat{\beta}_{KF.C}(t)L(t) \tag{33}$$

and the measurement error can be calculated by

$$\Sigma(t)L^2(t). \tag{34}$$

5. Simulation study

The simulation study uses district-wise data on the information of oilseeds production published by Ministry of Agriculture and Farmers Welfare, Department of Farmer Welfare and Directorate of Economics and Statistics to demonstrate the efficacy of the proposed method. The data provides information on oilseeds production areas in 236 districts of the ten states (Arunachal Pradesh, Uttarakhand, Nagaland, Jammu & Kashmir, Himachal Pradesh, Punjab, Uttar Pradesh, Bihar, Sikkim, and Assam) in India. The purpose of this study is to estimate the total/average area of oilseeds production in 2014. However, out of these 236 districts, there are 30 or more districts that produce oilseeds. Thus, the "area of oilseeds production" may be taken as a rare unit and the Adaptive Sampling design may be considered in this study. The estimation procedure, utilizing the KF technique more precisely predicts the total or average area of the oilseeds production in 2014, using the data of the past two years (2012 & 2013).

Therefore, considering the "area of oilseeds production" as a rare variable and "districts" as population units, the Adaptive Sampling technique is performed at the beginning. For this, an initial sample (a few districts) is selected and the values of the above mentioned variable are observed. If any of the districts produces oilseeds, its neighboring districts are also observed for further detection of a district which produces oilseeds. As a result, networks are formed and an estimator is defined for this sampling strategy. Here, the "number of agricultural laborers" is taken as an auxiliary variable from census 2011 data to employ the Greg estimator. Two distinct networks are observed in the population. The term "network" is defined in Sec. 3. Network sizes are different for each year. For instance, network sizes were 16 and 10 for the year 2012. For the year 2013, it was 17 and 8. In 2014, the network sizes are 28 and 7. Here, to study the competitiveness and effectiveness of the proposed model, it is assumed that the population total is known. The total areas (in Hectare) under the oilseeds crops are 33,563 (mean = 142.2161), 33,669 (mean= 142.6653) and 102,858 (mean= 435.839) in the year 2012, 2013 and 2014 respectively. In 2014, the number of districts as well as the total or average area, under the oilseeds crops is a bit larger than the previous years. In this paper, we concentrate on the estimation of average oilseed production rather than total production.

The simulation study is performed here selecting different sets of initial samples size. In Table 2 and Table 3, the initial samples sizes for the years 2012, 2013, and 2014 are shown as (67,77,87) or (58,60,74) or (63,65,78). Table 2 shows the performance report for the proposed method employing Thompson's approach where the initial samples are drawn by SRSWOR. The Greg estimate and its mean square error (MSE) estimate have been calculated for each year. These values are considered as initial estimates of the Kalman Filter's parameters. It is a recursive procedure of updating the existing estimator after adding the information gathered from the previous years. Therefore, it should have initial values which is stated in Eq. (12.1)

Table 1. Computation steps to evaluate the proposed estimators.

Kalman Filter with Thompson's Adaptive Sampling		Kalman Filter with Chaudhuri's Adaptive Sampling	
Target to evaluate	$t^{KF}_{greg}(t)$ (Eq.21)	Target to evaluate	$t^{KF}_{greg.C}(t)$ (Eq.33)
Previous Estimators for time points 0,1,… T	$t_{greg}(t)$, $v_{greg}(t)$ $\forall\ t = 0,1,\ldots T$ (Eqs. 6 − 7)	Previous Estimators for time points 0, 1, .T	$t_{greg.C}(t)$, $v_{greg.C}(t)$ $\forall\ t = 0,1,\ldots T$ (Eqs.23 − 24)
Computation Steps	1. Compute $A^*(t)$ (Eq.9) $\forall\ t = 0,1,\ldots T$ 2. Initial values at $t = 0$, $\varphi(0) = \frac{t_{greg}(0)}{X(0)}$ $\Sigma(0) = \frac{v_{greg}(0)}{X^2(0)}$ (Eq.12*a*) 3. $R(t) = \Sigma(t-1) + A^*(t)\ \forall\ t = 1,2,..T$ 4. $\Sigma(t)\ \forall\ t = 1,2\ldots T$ (Eq.19.2) 5. $t^*_{greg}(t) = \varphi(t-1)X(t)$, $\Delta(t), \varphi(t), \forall\ t = 1,2\ldots T$ (Eqs. 14, 19.1) 6. $\hat{\beta}_{KF}(t)$ $\forall\ t = 1,2\ldots T$ (Eq.20) 7. Updated estimator at $t = 1,2\ldots T$ $t^{KF}_{greg}(t)$ $\forall\ t = 1,2\ldots T$ (Eq.21)	Computation Steps	1. Compute $A^{**}(t)$ $\forall\ t = 0,1,\ldots T$ 2. Initial values at $t = 0$, $\varphi^*(0) = \frac{t_{greg.C}(0)}{L(0)}$ $\Sigma^*(0) = \frac{v_{greg.C}(0)}{L^2(0)}$ 3. $R^*(t)\ \forall\ t = 1,2,..T$ (Eq.28) 4. $\Sigma^*(t)\ \forall\ t = 1,2,..T$ (Eq.29) 5. $t^*_{greg.C}(t), \Delta^*(t), \varphi^*(t)$ $\forall\ t = 1,2\ldots T$ (Eqs. 30 − 32) 6. $\hat{\beta}_{KF.C}(t)$ $\forall\ t = 1,2\ldots T$ (Eq. 27) 7. Updated estimator at $t = 1,2\ldots T$ $t^{KF}_{greg.C}(t)$ $\forall\ t = 1,2\ldots T$ (Eq. 33)

of Sec. 3. In this study, the year 2012 is the initial time point i.e. $t = 0$. Thus, $\varphi(0) = \frac{t_{greg}(0)}{X(0)} = \frac{Greg\ \ estimate\ \ for\ \ year\ \ 2012}{X(0)}$ and $\Sigma(0) = \frac{v_{greg}(0)}{X^2(0)} = \frac{MSE\ \ of\ \ greg\ \ estimator\ \ for\ \ year\ \ 2012}{X^2(0)}$ are the initial values. The recursive algorithm steps referred to the Sec. 3 are followed to update the estimated value. For ease of understanding, the recursive steps are also provided in Table 1 shortly before this section.

Performances of the algorithms are tabulated in the following table with four basic criteria - Average Coefficient of Variation (ACV), Average Coverage Probability (ACP), Absolute Relative Bias (ARB), Pseudo mean square error (PMSE). For this, 10,000 replications of samples are taken. ACV is the average of "B" times replicated coefficient of variance (CV), which is formulated by $\frac{1}{B}\sum\frac{\text{Square root of estimated MSE}}{\text{Estimate}} \times 100$. 95% confidence interval (CI) is computed for "B" replicates and whether the CI covers the known value of the parameter or not is also noted. The percentage number of times the CI's cover the known value is called ACP. The closer it is to 95% the better. To calculate ARB, the absolute deviation of the estimate from the known value is taken relative to the known value for each replicate, and the average is considered. PMSE is the average square deviation of the estimator from the original. Generally, ACV less than 10% is an excellent estimator and at most 30% is acceptable. The smaller ARB indicates greater effectiveness. A similar conclusion can be drawn for PMSE.

Table 3 also shows the performance report for Kalman filtering employing Chaudhuri's Adaptive Sampling design. According to this approach, the surveyor draws the initial sample with a varying probability sampling design. Lahiri (1951)–Midzuno (1951)–Sen (1953)'s sampling design has been used here for this purpose which required a size measure variable. In this study, the "number of cultivators" is considered as a size measure variable that is taken from census 2011 data.

From Table 2, average of 10,000 times replicated greg estimates $(t_{greg}(t))$ with sample sizes 67, 77 and 87 are 139.13, 134.52 and 494.86 for the years 2012, 2013 and 2014, and the

Table 2. Generalized regression model vs. Kalman Filtering model (Thompson's approach).

Estimators	Performances based on 10,000 replications	Initial samples sizes for the years (2012,2013, 2014)		
		(67,77,87)	(58,60,74)	(63,65,78)
Using GREG	Average estimates	(139.13,134.52,494.86)	(144.28,147.53,431.86)	(145.92,146.19,443.06)
	Average MSE	(419.43,463.98,3329.88)	(438.73,527.66,3397.05)	(401..31,512.53,3390.32)
	ACV (%)	(14.20, 16.49, 11.49)	(14.00,16.38, 12.76)	(13.87, 16.45, 12.25)
	ACP	(84.48, 87.04, 74.89)	(85.06, 89.75, 79.56)	(85.93, 87.63, 78.95)
	ARB	(0.022,0.057,0.135)	(0.015,0.034,0.009)	(0.026,0.025,0.017)
Proposed estimator using KF	Average estimates	(–, 143.07,446.58)	(–,145.48, 432.02)	(–, 143.09, 434.92)
	ACV(%)	(–, 15.51, 10.68)	(–, 15.43, 10.06)	(–, 15.52, 11.39)
	ACP	(–, 92.83, 85.13)	(–, 93.74, 89.58)	(–, 92.81, 89.21)
	ARB	(–, 0.003,0.024)	(–, 0.019, 0.008)	(–, 0.003, 0.004)
	PMSE	(–, 1054.47, 9672.93)	(–, 1089.66, 5986.66)	(–, 1077.44, 7665.31)

Here Population mean $(2014) = 435.839$.

Table 3. Generalized regression model vs. Kalman Filtering model (Chaudhuri's approach).

Estimators	Performances based on 10,000 replications	Initial samples sizes for the years (2012,2013, 2014)		
		(67,77,87)	(58,60,74)	(63,65,78)
Using GREG	Average estimates	(124.25,122.58,445.63)	(159.65,157.64,406.48)	(131.25,126.61,488.63)
	Average MSE	(377.79,462.93,2955.58)	(455.40,559.72,2750.10)	(392.32,475.21,3189.24)
	ACV (%)	(15.48, 17.57, 12.18)	(13.18,14.90, 12.87)	(14.92, 17.15, 11.54)
	ACP	(89.31, 89.04, 75.80)	(90.37, 89.75, 73.56)	(89.21, 88.69, 75.06)
	ARB	(0.126,0.148,0.022)	(0.122,0.105,0.067)	(0.077,0.113,0.121)
Proposed estimator using KF	Average Estimates	(–, 141.33,436.58)	(–,143.22, 432.02)	(–, 141.43, 437.92)
	ACV (%)	(–, 15.54, 11.39)	(–, 14.22, 11.29)	(–, 15.86, 10.97)
	ACP	(–, 91.35, 85.17)	(–, 92.77, 84.54)	(–, 91.16, 85.06)
	ARB	(–, 0.009,0.002)	(–, 0.004, 0.009)	(–, 0.009, 0.005)
	PMSE	(–, 1051.45, 9113.07)	(–, 1059.66, 7876.59)	(–, 1007.38, 10885.31)

Here Population mean $(2014) = 435.839$.

corresponding average MSEs $(v_{greg}(t))$ are 419.43, 463.98 and 3329.88. The Greg estimate and its MSE (for each replication) are used as past information to update the estimate (for each replication) of population total for 2014. Thus, we get the updated average estimate of population mean as 446.58 using KF technique for 2014. To get KF estimate for 2014, we start with $t_{greg}(2012)$, $v_{greg}(2012)$ as an estimate and its variance estimate respectively.Therefore, the initial values of the parameters $(\varphi(0), \Sigma(0))$ of the proposed KF estimator are $(\frac{t_{greg}(2012)}{X(2012)}, \frac{v_{greg}(2012)}{X^2(2012)})$. Here $X(2012) = 21888998$ and $(t_{greg}(2012), v_{greg}(2012))$ values are different for each replications. Thus, it remains blank in that table in place of "Proposed estimator of KF". The information for the year 2012 first updates the estimate for the year 2013 with the help of KF technique described in Sec. 3, and then for the year 2014. ACV and ACP values are also mentioned here for Greg estimator and for the proposed KF estimator. The later one provide better coverage than the Greg estimator. KF estimator also provide lower ACV and ARB than the Greg estimator. Here we also get apparently same conclusion for other two sets of sample sizes.

In Table 3, we consider same sets of sample sizes. We reach to the same conclusion as above based on ACP, ACV and ARB values. To update the estimate of population mean for the year 2014, we have taken the information of the year 2012 i.e. $(t_{greg.C}(2012), v_{greg.C}(2012))$ and the initial values of the parameters $(\varphi^*(0), \Sigma^*(0))$ proposed KF estimator are $(\frac{t_{greg.C}(2012)}{L(2012)}, \frac{v_{greg.C}(2012)}{L^2(2012)})$ and $L(2012) = 24328982$.

6. Concluding remarks

There are large numbers of probabilistic sampling techniques that can be used to draw a sample from the target population. While dealing with the rare and clustered characteristics of a population, Adaptive Sampling is the best choice to provide a reliable estimate. This study further highlights the importance of periodic surveys of such populations to ensure their long-term existence. Here, a model-assisted estimator- Greg estimator is used for primary estimation. Also, the effects of regression coefficient and the auxiliary information on the estimation of the population total are observed and these effects are further considered for accuracy, utilizing the Kalman Filtering technique.

Therefore, the Kalman Filtering technique is applied here with the adaptive sampling. From simulation study, it is observed that proposed estimators improves the primarily suggested estimates, using past data and produced small absolute relative bias (ARB). This approach can track the rapidly changing process parameters. In this case, the regression coefficient is only the process parameter. As a result, survey practitioners may be interested in this proposed approach when dealing with rare and clustered characteristics.

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